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DIOPHANTINE ANALYSIS.

122. **Proposed by L. E. DICKSON, Ph. D., The University of Chicago.**

If p is a prime $(p^4-1)(p^2-1)$ has no factor of the form $1+p^3x$, $x>0$, if $p>2$; $(p^6-1)(p^4-1)(p^2-1)$ has no factor of the form $1+p^5x$, $x>0$.

No satisfactory solution has been received.

MISCELLANEOUS.

145. **Proposed by H. F. MacNEISH, Chicago, Ill.**

Two complete 5-plane configurations in space having the same vertices are identical; in general two complete $(n+2)$-faces in n-space having the same vertices are identical.

Solution by the PROPOSER.

We proceed at once to the general case.

Definition. An $(n+2)$-face in n space is defined as the n-space configuration formed by $(n+2)$ $(n-1)$-spaces and the intersections with the restriction that no $i+2$ $(i+j)$-spaces have a common j-space.

Suppose one $(n+2)$-face has the following notation: The $(n-1)$-spaces are specified $A^{n-1}_{i_r}$ $(r=1 \ldots\ldots n+2)$, and any j-space has notation of the type $A^j{}_{i_1 \ldots\ldots i_{n-j}}$ and the incidence relations are fully specified by stating that $A^j{}_{i_1 \ldots\ldots i_{k-j}}$ lies in precisely every element of higher dimensions whose subscripts are all of the set $i_1 \ldots\ldots i_{k-j}$.

(I) *Any three collinear points are of the type*

$$A'_{i_1 i_2 \ldots\ldots i_{n-1}} = A^0{}_{i_1 i_2 \ldots\ldots i_n}\ A^0{}_{i_1 i_2 \ldots\ldots i_{n-1}\, i_{n+1}}\ A^0{}_{i_1 i_2 \ldots\ldots i_{n-1}\, i_{n+2}}.$$

We consider any two points with every possible third point and show that this type is the only possible type of three collinear points.

(a) $A^0{}_{i_1 i_2 \ldots\ldots i_n}$, $A^0{}_{i_1 i_2 \ldots\ldots i_{n-1}\, i_{n+1}}$ $(n-1$ subscripts common). With these two points $A^0{}_{i_1 i_2 \ldots\ldots i_{n-1}\, i_{n+2}}$ is collinear from the $(n+2)$-face $A^{n-1}_{i_r}$; any other third point will be of the type $A^0{}_{i_2 \ldots\ldots i_{n+1}}$, which will then be on the line $A'_{i_1 i_2 \ldots\ldots i_{n-1}}$ (being collinear with $A^0{}_{i_1 i_2 \ldots\ldots i_n}$ and $A^0{}_{i_1 i_2 \ldots\ldots i_{n-1}\, i_{n+1}}$ in $(n+2)$-face $A^{n-1}_{i_r}$) and then $n+1$ $(n-1)$-space $A^{n-1}_{i_1}$, $A^{n-1}_{i_2} \ldots\ldots A^{n-1}_{i_{n+1}}$ will pass through $A^0{}_{i_2 \ldots\ldots i_{n+2}}$ which contradicts the definition of an $(n+2)$-face.

(b$_1$) $A^0{}_{i_1 i_2 \ldots\ldots i_n}$, $A^0{}_{i_3 i_4 \ldots\ldots i_{n+2}}$ $(n-2$ subscripts common) with $A^0{}_{i_1 i_2 \ldots\ldots i_{n-2}\, i_{n+1}\, i_{n+2}}$.

Now the line determined by $A^0{}_{i_1 i_2 \ldots\ldots i_n}$, $A^0{}_{i_3 i_4 \ldots\ldots i_{n+2}}$ lies in the plane $A^2{}_{i_3 \ldots\ldots i_n}$, and the line determined by $A^0{}_{i_1 i_2 \ldots\ldots i_n}$, $A^0{}_{i_1 i_2 \ldots\ldots i_{n-2}\, i_{n+1}\, i_{n+1}}$ lies in the plane $A^2{}_{i_2\text{----}i_{n-1}}$, and the line determined by $A^0{}_{i_3 i_4\text{----}i_{n+2}}$, $A^0{}_{i_1 i_2\text{----}i_{n+2}\, i_{n+1}\, i_{n+2}}$ lies in the plane $A^2{}_{i_3\text{----}i_{n-2}\, i_{n+1}\, i_{n+2}}$.

Therefore if $A^0{}_{i_1 i_2\text{----}i_n}$, $A^0{}_{i_3 i_4\text{----}i_{n+2}}$, $A^0{}_{i_1 i_2\text{----}i_{n-2}\, i_{n+1}\, i_{n+2}}$ are collinear, these three planes are coaxial, which contradicts the definition.

(b_2) $A^0{}_{i_1 i_2\text{----}i_n}$ $A^0{}_{i_3 i_4\text{----}i_{n+2}}$ with $A^0{}_{i_2\text{----}i_{n+1}}$. On this line will lie the point $A^0{}_{i_2\text{----}i_n i_{n+2}}$ for it is collinear with $A^0{}_{i_1 i_2\text{----}i_n}$ and $A^0{}_{i_2\text{----}i_{n+1}}$ in the $(n+2)$-face $A^{n-1}_{i_r}$ and at $A^0{}_{i_3\text{----}i_{n+2}}$ we then have $n+1$ (-1)-spaces concurring, which contradicts the definition.

Therefore the points are collinear only according to type (a); there are exactly ${}_{n+2}C_{n-1}$ such lines, and these are the lines of the $(n+2)$-face $A^{n-1}_{i_r}$.

Therefore if the xertices of two $(n+2)$-faces are identical the lines are identical.

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(II) *Any $k+3$ k-spaces in the same $(k+1)$-spaces are of the type for $k=0$ ……… $n+2$.*

$$\text{(a)}\quad \left.\begin{array}{l} A^{k+1}_{i_1\text{----}i_{n-k-1}} = A^k{}_{i_1\text{----}i_{n-k-1}\, i_{n-k}} \\ \qquad\qquad\quad A^k{}_{i_1\text{----}i_{n-k-1}\, i_{n-k+1}} \\ \qquad\qquad\quad A^k{}_{i_1\text{----}i_{n-k-1}\, i_{n-k+2}} \\ \qquad\qquad\quad \cdots\cdots\cdots\cdots \\ \qquad\qquad\quad \cdots\cdots\cdots\cdots \\ \qquad\qquad\quad A^k{}_{i_1\text{----}i_{n-k-1}\, i_{n+2}} \end{array}\right\}$$

For if only $n-k-2$ or less subscripts are common for any two k-spacs in the same $(k+1)$-space we would have k-spaces of the types $A^k{}_{i_1\text{----}i_{n-k-2} i_l i_m}$, $A^k{}_{i_1\text{----}i_{n-k-2} i_r i_s}$ lying in the same $(k+1)$-space. They intersect in a $k-1)$-space which will lie in $(n-k+2)$ $(n-1)$-spaces; *i. e.*, $A^{n-1}_{i_1}$, $A^{n-1}_{i_2}$, ………, $A^{n-1}_{i_{n-k-2}}$, A^{n-1}_{il}, A^{n-1}_{im}, A^{n-1}_{ir}, A^{n-1}_{is}, which contradicts the definition that no $i+2$ $(i+k)$-spaces have a common k-space.

Therefore if the k-spaces of two $(n+2)$-faces are identical the $(k+1)$-spaces are identical $k=0$, ………, $n+2$.

Therefore the two $(n+2)$-faces are identical throughout.